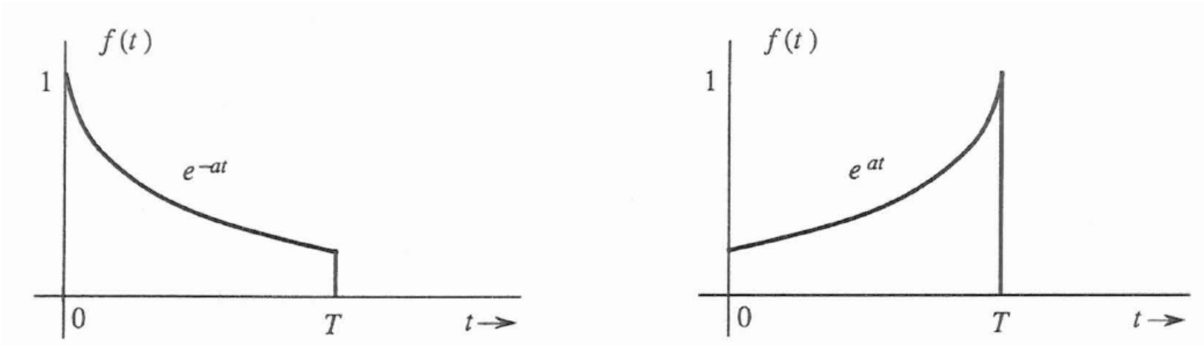
E2.5 Signals & Linear Systems

Tutorial Sheet 6 – Fourier Transform

(Lectures 10 - 11)

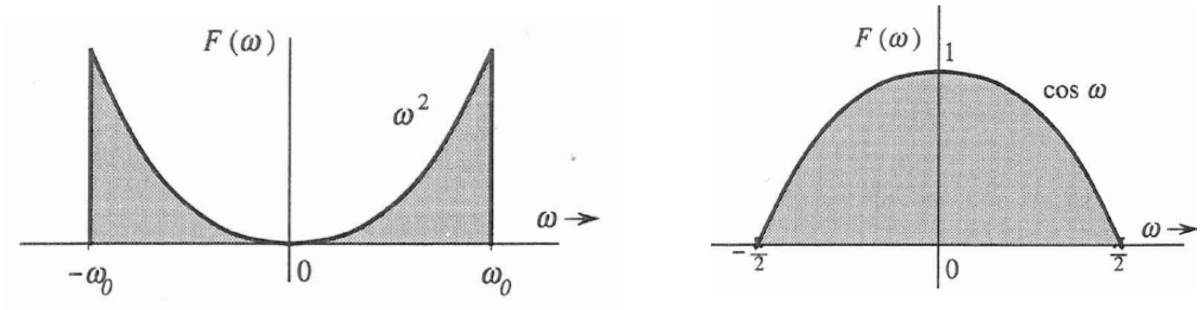
1.* Derive the Fourier transform of the signals *f(t)* shown in Fig. Q1 (a) amd (b).



(a) (b)

Figure Q1

2.* Derive the inverse Fourier transform of the spectra shown in Fig. Q2 (a) and (b).



(a) (b)

Figure Q2

3.* Sketch the following functions:

a) $rect(\frac{t}{2})$ b) $rect(\frac{t-10}{8})$

c) $\text{sinc}(\frac{\pi\omega}{5})$ d) $\text{sinc}\,(\frac{\omega-10\pi}{5})$.

4.** Apply the duality property to the appropriate of the Fourier Transform table (Lec 10, slides 13-15), show that:

a) $\frac{1}{2}[\delta(t)+\frac{j}{\pi t}] \Leftrightarrow u(\omega)$

b) $\frac{1}{t} \Leftrightarrow -j\pi\ \text{sng}(\omega)$

c) $\delta(t+T)-\delta(t-T) \Leftrightarrow 2j\sin(T\omega)$

5.** The Fourier transform of the triangular pulse *f(t)* shown in Fig. Q5(a) is given to be:

$$F(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1)$$

Use this information and the time-shifting and time-scaling properties, find the Fourier transforms of the signals $f_1(t)$ to $f_5(t)$ shown in Fig. Q5 (b)-(f).

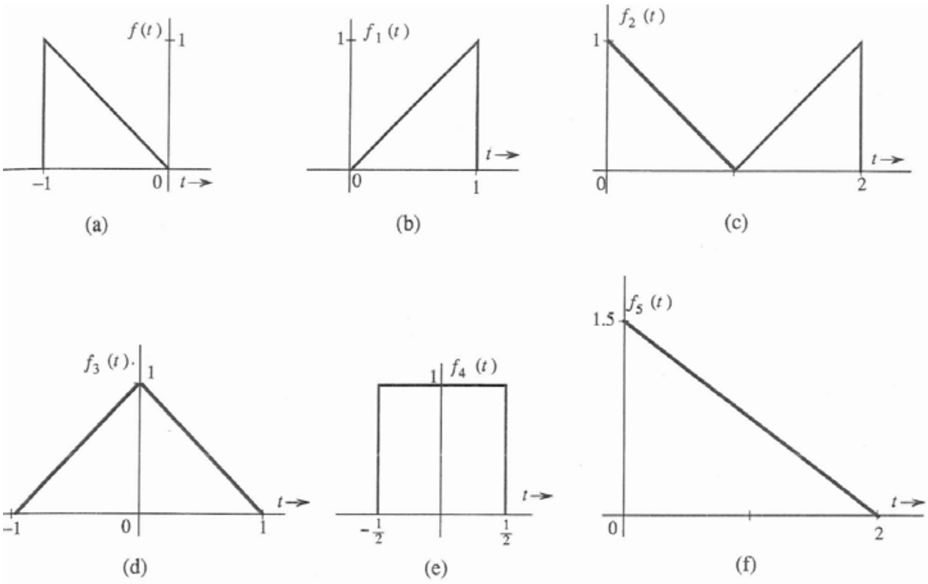


Fig. Q5

6.** The signals in Fig. Q6 (a)-(c) are modulated signals with carrier cos 10t. Find the Fourier transforms of these signals using appropriate properties of the Fourier transform and the FT table given in Lecture 10, slides 13-15. Sketch the amplitude and phase spectra for (a) and (b).

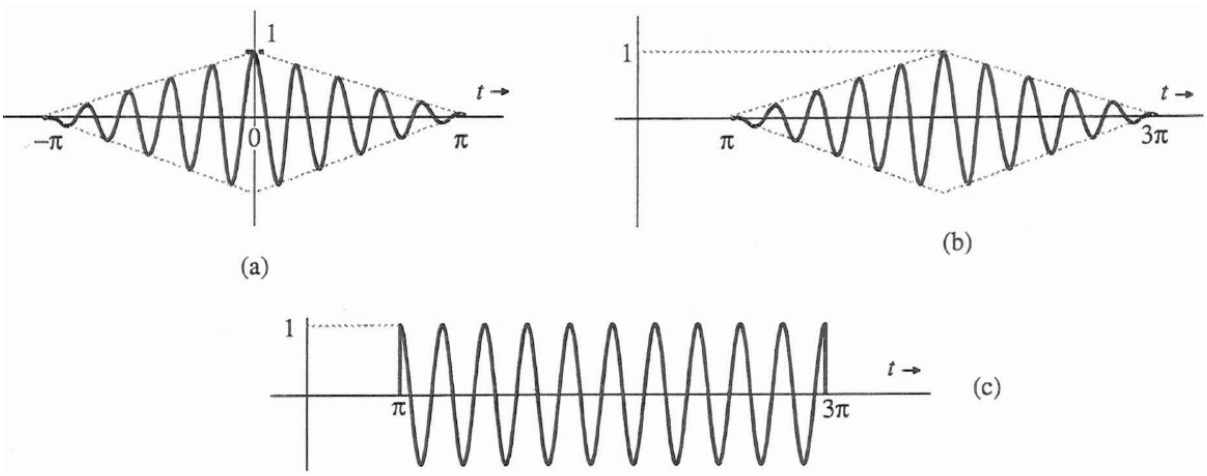


Fig. Q6

7.*** The process of recovering a signal *f(t)* from the modulated signal $f(t)\cos\omega_0 t$ is called **demodulation**. Show that the signal $f(t)\cos\omega_0 t$ can be demodulated by multiplying it with $2\cos\omega_0 t$ and passing the product through a lowpass filter of bandwidth W radians/sec. Assume that W < ω.

8.*** Derive from first principle the Fourier transform of the unit step function *u(t)*.